A Performance Model for Single Charger Multi-Socket Charging Stations in Shared Mobility Systems

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***Abstract*—It is commonly recognised that the availability of more pervasive networks of public fast-charging stations is a key incentive for the market growth of electric vehicles (EVs). In order to reduce the huge capital investments that are needed for the deployment of such charging infrastructure, new types of charging facilities have been proposed, which allow to charge several EVs simultaneously by sharing the resources of a single charger over multiple co-located charging sockets. To characterise the behaviour of a single-charger multiple-socket (SCMS) system under stochastic EV charging demands, in this work we propose a continuous time and discrete state space Markov chain model. Our analytical model applies to scenarios in which the duration of charging periods is uncertain. This typically occurs in *shared* mobility systems (e.g., car sharing services) in which customers randomly arrive at the charging station to pick up available shared EVs. We examine two scenarios of increasing complexity. In the first one, customers can pick up only fully-charged EVs. In the second scenario, EVs can leave the station before the charging process is complete. Our numerical results assess the impact of station capacity (both physical space and grid connection) on the system performance from the perspective of both the customers and the infrastructure owner.**

I. Introduction

The accelerated development of new models of electric cars, lower battery prices, and government incentives are actively supporting electric car market deployment, and sales of new EV worldwide surpassed 1 million units in 2017 [1]. However, EV market share is still much lower than expected. The major barriers to EV market growth are the access to a sufficiently pervasive charging infrastructure, and the speed of charging [2]. Therefore, cities are taking great efforts for deploying large on-street infrastructures of fast charging stations, which provide the possibility to fully charge an EV battery in less than half an hour [3].

To increase the utilisation of high-cost charging stations and to reduce the corresponding capital investments, new types of charging facilities have been recently proposed with the capability of sharing charging resources to multiple charging sockets [4], [5]. Specifically, charging stations implementing a single charger can be equipped with multiple charging sockets and a load management system to enable sharing the charging power with multiple connected EV simultaneously. Intuitively, coordinated charging enhances the utilisation of the charging station as several EVs can share the same charger, and charging resources are not wasted if an EV is completely charged but it still occupies a parking space. Furthermore, power sharing helps to flatten charging loads during peak periods with respect to the case in which each parking space is served by a separate charger [6].

The planning and control of a charing infrastructure require the availability of suitable models to characterise the behaviour of the charging stations. However, the majority of existing models assume a fixed charging power [7], and they do not apply to stations using power sharing. The second major challenge is that most previous studies focus on private EVs and destination charging. Specifically, the driver of the EV is assumed to be the exclusive owner of the vehicle and to recharge the vehicle battery at the final destination of his journey (e.g. home, workplace). In this case the parking time (e.g., night, working shift) is usually longer than the required charging time and the vehicle is fully recharged when departing from the charging station. However, transportation services relaying on shared fleets of electric vehicles, such as electric car sharing[1], are becoming increasingly popular in urban areas. In this case, the parking time is not known in advance as it depends on the arrival pattern of customers at the car sharing station. This complicates the modelling of the charging load profile as the charging service could be interrupted at any time when the customer arrives at the station to pick up one of available shared vehicles.

To address the above challenges, this paper develops a stochastic model based on a discrete Markov chain to explain the behaviour of charging stations using a power sharing discipline, accounting for random arrival patterns of both vehicles and EV drivers. We examine two scenarios of increasing complexity. In the first one, customers can pick up only fully-charged EVs. In the second scenario, EVs can leave the station before the charging process is complete. The numerical results obtained through realistic simulations and the steady state solution of the proposed model confirm the validity of the modelling approach. Furthermore, we conduct a sensitivity analysis of various metrics of interest, such

[1]https://www.evcard.com/.

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as blocking probabilities due to lack of parking spaces or fully-charged vehicles, and the power utilisation. Finally, our numerical results also disclose the ability of power sharing technologies to improve the cost-effectiveness and reliability of a shared electrical mobility service.

The remainder of this paper is organised as follows. Section II defines the charging scenarios. In Section III the analytical model for the charging station is developed. Numerical results are presented in Section IV. Section V concludes the paper and discusses future work.

II. Types of Charging Stations and Target Scenarios

Following the technology development of EVs, we are also witnessing a rapid evolution of EV charging infrastructures. The main focus for the development of charging technologies has been the speed of charging, but new evolutions are under development, including dynamic load management, V2G services, energy storage, inductive charging [8], [9]. A key trend in the market of charging stations is to build large charging facilities with multiple charging points (namely plugs), to facilitate the adoption of large EV fleets [1]. However, different options may substantially differ in terms of investment and operational costs, as well as management complexity. In the following, we overview the types of charging stations that are modelled in this work, which are also summarised in Table I. Then, we detail the types of charging scenarios that we consider for a shared mobility service.

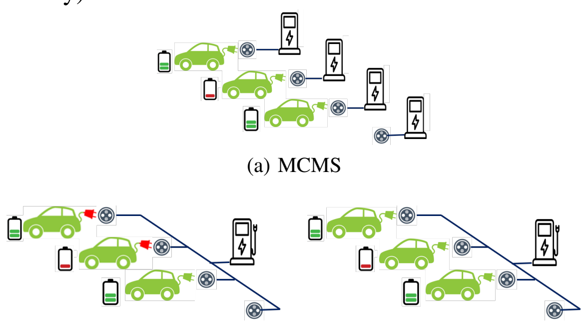
A. Charging Stations

The traditional charging station model for large charging facilities is the Multi-Charger Multi-Socket (MCMS) model (see Figure 1a), which consists of N charging sockets, each serving a certain parking space and connected to a separate charger to independently charge the parked EVs. Thus, an MCMS station can be regarded as a combination of multiple single charging piles. Clearly, the MCMS model is expensive to install and operate, as it requires a potentially large number of chargers and a connection to the power grid capable of sustaining up to N simultaneous charging processes, each draining a power P from the grid. Furthermore, the MCMS model is potentially inefficient, as the chargers are idle if a parking space is empty or the connected EV is fully charged. An alternative model, called Single Charger Multi-Socket (SCMS), is proposed in [4]. Similarly to the MCMS station, the SCMS station (see Figure 1b) has multiple sockets that can be connected to several EVs, but a single charger. Then, the SCMS station is equipped with a load switch that allows to charge only one of the plugged EVs at any time. Thus, EVs can enter the station if there are empty parking spaces, but they have to wait their turn to recharge. In this way, the infrastructure operator can save money by deploying a small number of expensive chargers and avoiding cost-intensive increases in grid connection capacity, while idle sockets can be used as waiting spots.

TABLE I
MAIN CHARACTERISTICS OF DIFFERENT TYPES OF CHARGING STATIONS

Type	# chargers	# served cars	# parked spaces	max power load
MCMS	N	N	N	$N \times P$
SCMS	1	1	N	P
SCMS-PS	1	N	N	P

A more flexible design, called Single Charger Multi-Socket-Power Sharing (SCMS-PS), is proposed in [5], [6] by leveraging the innovative concept of *power sharing* (see Figure 1c). Similarly to the SCMS model, a SCMS-PS station is equipped with a single charger but the load switch is substituted with a more sophisticated load management system, which allows to serve multiple EVs at the same time and to regulate the energy that is provided to each connected EV. Thus, the SCMS-PS model ensures almost the same economic advantages of the simpler SCMS system, but it also provides the capability of managing EV charging for various purposes, such as to leverage real-time electricity tariffs or to avoid grid overloading during peak traffic times. It is worth pointing out that the mobility operator can also exploit coordinated charging to improve the service provided to its customers. For instance, to prioritise the charging of the set of parked EVs with low SoC. Clearly, the charging performance of both SCMS and SCMS-PS models depend on both the parking profiles (namely, vehicle arrival distributions and parking duration distributions) and the charging policies. Moreover, in the case of shared EVs, the passengers' arrival distribution also plays a critical role. Hence, we aim to answer two main research questions: 1) which are the benefits of power sharing from the perspective of both the customers and the operator of a shared mobility system, and 2) how to size the charging station to achieve given performance bounds (e.g., bounded customers' blocking probability).



(a) MCMS

(b) SCMS (c) SCMS-PS

Fig. 1. Charging typologies

B. Charging Scenarios

Typically, two types of charging scenarios are considered in previous studies [4], [5]. The first one is known as *destination charging*, as it occurs when the EV driver reaches its intended destination (e.g, home, workplace). In this case, parking is the main purpose of the trip and parking durations are typically sufficient for fully charging the battery (e.g., overnight charging at home). The second type of charing scenario is denoted as *urgent charging*, as it occurs when the SoC goes below a given threshold and the EV driver makes a brief stopover to (partially) recharge the battery before continuing with his journey. In this case, the main purpose of the parking phase is charging and the EV leaves the charging station as soon as the SoC is regarded as sufficient to reach the final trip destination. Differently from previous studies, in this work we assume that charging stations are providing their charging services to a fleet of shared electric cars. In this case, shared vehicles are dropped off at a charging station and remain parked (and plugged) until a new customer arrives to pick up one of the available EVs at the station. As a consequence, parking duration is independent of the purpose of the last trip (different from destination charging scenarios), and charging sockets can remain occupied even if the battery is fully charged (different from urgent charging scenarios). Hence, in a shared mobility system neither destination charging nor urgent charging scenarios are appropriate. The types of charging scenarios we consider are the following:

- *Full Charge (FC) scenario*: In this scenario, a vehicle enters the station if it can immediately plug into a socket (i.e., there is an empty parking space). A new customer is allowed to pick up a vehicle only if it is fully charged, otherwise he/she is rejected. For such a system, the customers' blocking probability is a natural performance metric.
- *Immediate Pickup (IP) scenario*: In this scenario, a new customer who arrives at the station is always allowed to pick up a vehicle. The plugged shared vehicle with the highest SoC is assigned to the new customer. Clearly, this scenario is able to admit more customers than the previous one (a new customer is rejected only if the station is empty), and, thus, it is more profitable. However, the level of QoS provided to the customers degrades as the SoC of rented vehicles is potentially lower.

In the following section, we develop an analytical model of the SCMS-PS system for both the above scenarios, and we discuss how to adapt the model to simpler MCMS and SCMS systems.

III. Charging Station Stochastic Model

As in previous studies, the charging station is modelled as a continuous-time Markov chain with discrete state space [10], [11]. More formally, let N_c and N_f be two random discrete variables representing the number of plugged vehicles that are recharging and the number of plugged vehicles that are fully charged, respectively. Let n_c and n_f be two realisations of these random variables. Then, it holds that $(n_c + n_f) \leq N$ because the charging station has a limited number of parking spaces. We assume that shared vehicles are dropped off at the station according to a Poisson process with rate λ vehicles/hour. Similarly, we assume that the inter-arrival time of new customers at the station follows an exponential distribution with parameter μ customers/hour.

To model the charging process, some preliminary considerations must be made. First, vehicles arrive at the station with a random SoC, and a variety of different EV models with different battery sizes may exist[2]. To capture such variability, the analytical model assumes that the service demand, namely the time needed to recharge an EV battery using a constant power P is exponentially distributed with rate ν[3]. Furthermore, even if the power drained from the grid is constant the energy delivered to each plugged vehicle with power sharing depends on the number n_c. Under the assumption that the charging power is *equally* divided among the recharging vehicles, namely a Processor Sharing (PS) serving discipline is adopted [12], each recharging vehicle receives service at a rate ν/n_c. Given the exponential distribution of service demands, it holds that, independently of the number n_c, the overall probability per time unit that the service of some recharging vehicles ends is ν. As better explained in the following, this property greatly simplifies the model derivation.

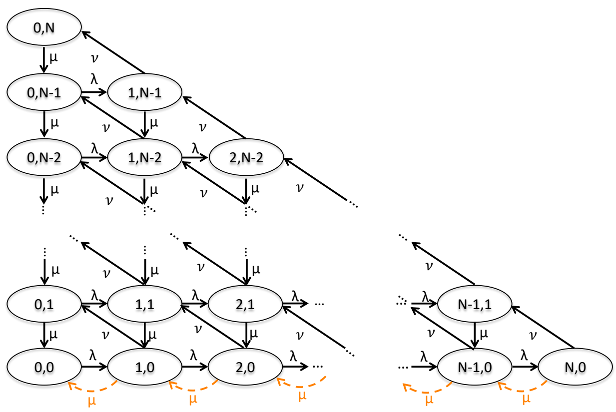


Fig. 2. Markov chain of a SCMS-PS station. The orange transitions exist only for the IP scenario.

Given the assumptions above, the state space of the Markov chain, along with its transmission rates can be fully characterised (see Figure 2). For the sake of notation brevity, let sc be a boolean value that labels the scenario considered, i.e. sc = FC when the Full Charge scenario is modelled and sc = IP if the Immediate Pickup scenario is modelled. Now, let us consider the states of the main diagonal of the Markov chain in Figure 2, namely states $(i, N-i)$, for $i = 0, \ldots, N$. These states are blocking states, which means that no vehicles can be admitted into the system. Beyond these blocking states, λ gives the transition rate from states (n_c, n_f) to states $(n_c + 1, n_f)$, with $n_c, n_f < N$, since all dropped vehicles go to charging mode. When a new customer arrives, the state transitions depend on the charging scenario according to the following conditions.

- *FC scenario*: If a fully charged vehicle is available at the station, i.e. $n_f > 0$, a transition to state $(n_c, n_f - 1)$ occurs with rate μ.

[2]For simplicity, in [11] it is assumed that vehicles arrive at the charging station with an empty battery and all battery sizes are equal.

[3]To confirm the validity of this exponential assumption in the simulations we use deterministic service times

- *IP scenario*: The arrival of a new customer leads to a transition to a state $(n_c, n_f - 1)$ if $n_f > 0$; otherwise to a state $(n_c - 1, 0)$ if $n_c > 0$ with rate μ.

Finally, a transition from states (n_c, n_f) to states (n_c+1, n_f-1) occurs with rate ν when the battery of a plugged vehicle becomes fully charged.

Let $\boldsymbol{\pi}^{(1,\mathrm{sc})}$ be the steady state distribution vector of the Markov chain in Figure 2 for the scenario sc. For the sake of notation brevity we assume that system states are ordered in anti-lexicographical order, namely $\boldsymbol{\pi}^{(1,\mathrm{sc})} = (\pi_{0,0}, \ldots, \pi_{0,N}, \pi_{1,0}, \ldots, \pi_{1,N-1}, \pi_{2,0}, \ldots, \pi_{N,0})$. Let $\mathbf{G}^{(\mathrm{sc})}$ be the generator matrix of the Markov chain for scenario sc. Then, it must hold that

$$\boldsymbol{\pi}^{(1,\mathrm{sc})}\mathbf{G}^{(\mathrm{sc})} = 0 \; , \tag{1}$$

and $\sum_{i=0}^{N}\sum_{j=0}^{N-i}\pi_{i,j}^{(1,\mathrm{sc})} = 1$.

Owing to the regularity of the Markov chain in Figure 2 it holds that

$$\mathbf{G}^{(\mathrm{sc})} = \left[\begin{array}{c|c|c|c|c|c|c|c} D_{N+1} & C_N & 0 & 0 & \ldots & 0 & 0 & 0 \\ \hline A_{N+1} & B_N^{\mathrm{sc}} & C_{N-1}^{\mathrm{sc}} & 0 & \ldots & 0 & 0 & 0 \\ \hline 0 & A_N & B_{N-1}^{\mathrm{sc}} & C_{N-2}^{\mathrm{sc}} & \ldots & 0 & 0 & 0 \\ \hline \vdots & & \ddots & \ddots & \ddots & & & \vdots \\ \hline 0 & 0 & 0 & 0 & 0 & A_3 & B_2^{\mathrm{sc}} & C_1^{\mathrm{sc}} \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & A_2 & B_1^{\mathrm{sc}} \end{array}\right], \tag{2}$$

where the scenario-dependent matrices A_n, D_n, B_n^{sc} and C_n^{sc} have dimensions $(n-1)\times n$, $n\times n$, $n\times n$, and $(n+1)\times n$, respectively (n is a parameter). For the sake of notation brevity, let I_n denote the $n\times n$ square identity matrix, and let $\underline{0}_n$ be an n-vector with all entries equal to 0. Then, we can write

$$A_n = \left(\begin{array}{c|c} -\lambda I_n & \underline{0}_n \end{array} \right) \qquad \forall n = 2 \ldots N+1 \tag{3}$$

$$D_n = \begin{pmatrix} \lambda & -\mu & 0 & \ldots & 0 \\ 0 & \lambda+\mu & -\mu & \ldots & 0 \\ \vdots & & \ddots & \ddots & \\ 0 & 0 & \ldots & \lambda+\mu & -\mu \\ 0 & 0 & \ldots & 0 & \mu \end{pmatrix} \qquad \forall n = 2, \ldots, N+1 \tag{4}$$

$$B_n^{FC} = D_n + \mu_c I_n \qquad \forall n = 2, \ldots, N \tag{5}$$

$$B_1^{FC} = \mu_c \tag{6}$$

$$B_n^{IP} = \begin{pmatrix} \lambda+\mu+\mu_c & -\mu & 0 & \ldots & 0 \\ 0 & \lambda+\mu+\mu_c & -\mu & \ldots & 0 \\ \vdots & & \ddots & \ddots & \\ 0 & 0 & \ldots & \lambda+\mu+\mu_c & -\mu \\ 0 & 0 & \ldots & 0 & \mu+\mu_c \end{pmatrix} \qquad \forall n = 2, \ldots, N \tag{7}$$

$$B_1^{IP} = (\mu + \mu_c) \tag{8}$$

$$C_n^{FC} = \left(\begin{array}{c} \underline{0}_n^T \\ \hline -\mu_c I_n \end{array} \right) \qquad \forall n = 1, \ldots, N \tag{9}$$

$$C_n^{IP} = \left(\begin{array}{c|c} \mu & \underline{0}_{n-1}^T \\ \hline \multicolumn{2}{c}{-\mu_c I_n} \end{array} \right) \qquad \forall n = 1, \ldots, N \tag{10}$$

After some algebraic manipulations, an exact solution of Equation (1) can be determined and the following closed-form expression is available for the FC scenario.

$$\pi_{n_c,n_f}^{(1,\mathrm{FC})} = \frac{\rho_c^{n_c}\rho_f^{n_f}}{d_1} \; , \tag{11}$$

where $\rho_c = \lambda/\mu$, $\rho_f = \lambda/\nu$ and d_1 is equal to:

$$d_1 = \frac{\rho_c + \rho_c^{(2+N)}(-1+\rho_f) - \rho_c\rho_f^{(2+N)} + \rho_f(-1+\rho_f^{1+N})}{(-1+\rho_c)(\rho_c-\rho_f)(-1+\rho_c)}. \tag{12}$$

Due to space constraints, the analytical models for the SCMS and MCMS systems are not fully described, but we discuss the main differences from the Markov chain in Figure 2. As far as the SCMS model is concerned, we can observe that in state (n_c, n_f) only one of the n_c vehicles is charging with a service rate ν, while the other $n_c - 1$ vehicles are waiting until they can be served. Since, one vehicle obtains the entire service capacity ν of the station, the transition from state (n_c, n_f) to state $(n_c - 1, n_f + 1)$, with $n_c + n_f < N$, occurs with rate ν. This implies that the the Markov chain of the SCMS model is identical to the Markov chain of the SCMS-PS model. Let $\boldsymbol{\pi}^{(2,\mathrm{sc})}$ be the steady state vector distribution of the SCSM model. It holds that $\boldsymbol{\pi}^{(2,\mathrm{sc})} = \boldsymbol{\pi}^{(1,\mathrm{sc})}$. This is an important result because it demonstrates that the SCMS-PS and SCMS models achieves the same average performance in steady state. However, there is a crucial difference between the SCMS-PS and SCMS models when we analyse the time average a vehicle spends in charging or waiting for charge given that it requires an amount of service x, say $E[D, x]$. In the case of an SCMS-PS system, we can assume that $E[D, x] = \omega x$, for some constant ω by analogy with what is known for M/M/1-PS queue [12]. On the other hand, in the case of an SCMS system, it holds that $E[D, x] = E[W] + x$, where $E[W]$ is the waiting time before other vehicles in the system have fully recharged (FC scenario) or have been picked up by a new customer (IP scenario). We are not able to compute an exact expression for the constant ω. However, we can observe that, if x is small (i.e., the battery of the arriving vehicle is almost full) the SCMS-PS station is to be preferred, while if x is large the SCMS system ensures smaller charging delays. In Section IV, we will provide evidence for this intuition.

As far as the MCMS model is concerned, it differs from the SCSM-PS model only for the rate of transitions from state (c, f) to state $(c - 1, f + 1)$. More precisely, since the MCMS system is equipped with N independent chargers, the rate of this transition is equal to $n_c\nu$. Then, we can solve the model and compute the steady state vector distribution $\boldsymbol{\pi}^{(3,\mathrm{sc})}$ using the same line of reasoning of Equation (1). For the FC scenario, it is also possible to derive the following closed form expression:

$$\pi_{n_c,n_f}^{(3,\mathrm{FC})} = \frac{\rho_c^{n_c}\rho_f^{n_f}}{n_c! d_2}, \tag{13}$$

where d_2 is equal to:

$$d_2 = \frac{\left(\frac{\rho_c}{\rho_f}\right)^{-N}\left(-e_f^{\rho}\left(\frac{\rho_f}{\rho_c}\right)^{N}\Gamma\left(1+N,\rho_f\right) + e^{\frac{\rho_f}{\rho_c}}\rho_c\rho_f^N\Gamma(1+N,\frac{\rho_f}{\rho_c})\right)}{(-1+\rho_c)\Gamma(1+N)}, \tag{14}$$

and $\Gamma(\cdot)$ is the Gamma function.

A. *Metrics of Interest*

Given the $\boldsymbol{\pi}^{(i,\mathrm{sc})}$ distribution, it is straightforward to derive a set of metrics to measure the behaviour of the charging station, and the level of QoS provided to the customers of the shared mobility system.

Probability of a vehicle being blocked $\left(p_{b,v}^{(i,\mathrm{sc})}\right)$: The probability of a vehicle being rejected because there are not empty parking spaces. It is straightforward to write:

$$p_{b,v}^{(i,\mathrm{sc})} = \sum_{k=0}^{N} \pi_{k,N-k}^{(i,\mathrm{sc})} , \tag{15}$$

with $i = 1$ for the SCMS-PS system, $i = 2$ for the SCMS system, and $i = 3$ for the MCMS system.

Probability of a customer being rejected $\left(p_{b,c}^{(i,\mathrm{sc})}\right)$: The probability of a customer being rejected because the charging station is empty (IP scenario) or there is not a fully charged vehicle (FC scenario). We can write that:

$$p_{b,c}^{(i,\mathrm{FC})} = \sum_{k=0}^{N} \pi_{k,0}^{(i,\mathrm{FC})} \tag{16a}$$

$$p_{b,c}^{(i,\mathrm{IP})} = \pi_{0,0}^{(i,\mathrm{IP})} \tag{16b}$$

Distribution of power consumption: Let $Q^{(i,\mathrm{sc})}$ be the a random variable representing the power drained by the charging station from the grid under the scenario sc. Clearly, a different charging power corresponds to the different states of the Markov chain. Then, the probability that $Q^{(i,\mathrm{sc})} = q_l$, with $q_l \in [0, NP]$, is the sum of the probabilities of the set of states for which the charging station provides such power to the plugged station. Thus, we can write that

$$P\{Q^{(3,\mathrm{sc})} = kP\} = \sum_{i=0}^{N} \pi_{k,i}^{(3,\mathrm{sc})} \quad \forall k = 0, \ldots, N \tag{17}$$

$$P\{Q^{(1/2,\mathrm{sc})} = kP\} = \begin{cases} \sum_{l=0}^{N} \pi_{k,l}^{(1/2,\mathrm{sc})} & k = 0 \\ 1 - P\{Q^{(1/2,\mathrm{sc})} = 0\} & k = 1 \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

Note that from the distribution of power consumptions, it is easy to derive other statistics, such as the expected normalised power consumption, say $E[P^{(i,\mathrm{sc})}] = \sum_{k=1}^{N} kP \times P\{Q^{(i,\mathrm{sc})} = kP\}$, or the power utilisation, say $E[P_{\%}^{(i,\mathrm{sc})}]$, defined as the ratio between the expected normalised power consumption and the normalised power capacity of the station. It is straightforward to observe that:

$$E[P_{\%}^{(i,\mathrm{sc})}] = \begin{cases} E[P^{(i,\mathrm{sc})}]/NP & i = 1 \\ E[P^{(i,\mathrm{sc})}]/P & \text{otherwise} \end{cases} \tag{19}$$

We can observe that in the IP scenario, all the states of the i-the diagonal, i.e. $D_i = \{(c, f) : c + f = i\}$ are connected with all the states of the $(i+1)$-th diagonal by outgoing edges of rate λ and ingoing edges of rate μ. This means that the two-dimensional Markov Chain behaves in the diagonal states D_i as an M/M/1/N queue. Thus for the IP scenario we obtain that:

$$p_{block}^{IP} = \rho^N \frac{1-\rho}{1-\rho^{M+1}} \tag{20}$$

$$p_{rej}^{IP} = \frac{1-\rho}{1-\rho^{N+1}}. \tag{21}$$

This result is particularly interesting because tells us that in this scenario, there are no advantages on choosing one technology or another from an operational point of view (at least for the probability to find a full station or an empty station). This definitely confirms that the MCMS is unprofitable, because MCMS is more expansive in terms of power consumption and it does not provide better vehicle availability. In the following section we will discuss these points more extensively.

IV. Performance Evaluation

In this section we present numerical results to validate the accuracy of the modelling approach, and we perform a sensitivity analysis of different metrics with respect to model parameters. Average statistic and 95% confidence intervals are computed by replicating ten times each simulation run, which simulates 10^6 customers' arrival events. Confidence intervals are very narrow and they do not appear in the following graphs.

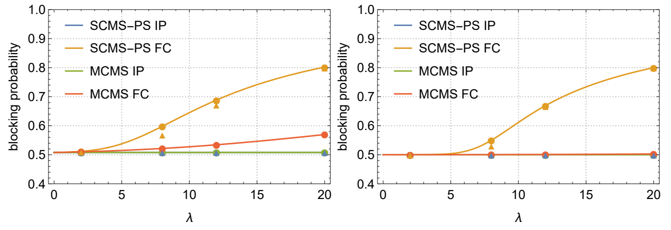
A. Parameter Settings

The model parameters used in the evaluation are motivated by [11], [13]. Specifically, the vehicle arrival rate varies from 2 to 20 vehicles/hour. Two station sizes are considered, namely $N = 5$ and $N = 10$. We suppose that each EV has a battery of 24 kWh, which is the battery of Nissan Leaf, and that an EV entering a charging station has a SoC that is uniformly sampled over the interval $[0.2, 0.8]$. Under this setting, a fast-charging station (i.e., with a charging power P equal to 50KW) gives an average service rate ν equal to 2 vehicles/hour. Regarding the customers' arrival rate

B. Results

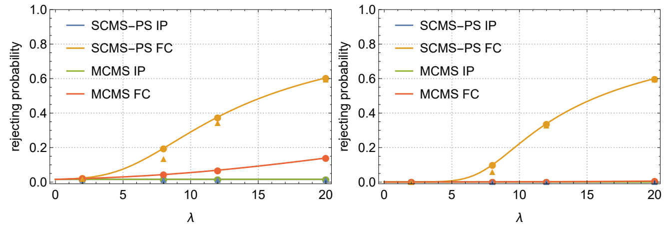
1) Mobility operator perspective: Figure 3 compares the blocking probability of vehicles versus the λ rate for the different charging station models and charging scenarios. The numerical results are obtained considering a medium utilisation of the transportation service with $\mu = \frac{\lambda}{2}$. First of all, we can observe that the simulations results corroborate the model predictions. This confirms that the exponential service assumption is adequate to model also a constant power charging. A second observation is that the blocking probability is constant in the IP scenario. This is due to the fact that the ratio between λ and μ is fixed. On the other hand, we can see that in the FC scenario, the larger λ and the higher the blocking probability. This can be explained by noting that a higher λ means a higher number of vehicles that need to be fully charged, which results into an increase of the charing time. As expected, this behaviour is less critical in the MCMS-FC scenario than in SCMS and SCMS-PS due to the higher power capacity. Interestingly, we can also note that the size of the charging station has no impact on the performance of the SCMS and SCMS-PS cases because in this setting (i.e., fixed ratio between λ and μ) the blocking probability is primarily correlated to the recharging speed.

In Figure 4, we investigate the impact of customers' arrival rates on the blocking probability for $\lambda = 15$ vehicles/hour and $N = 10$. Interestingly we can observe that the blocking



(a) $N = 5$ (b) $N = 10$

Fig. 3. Blocking probability versus vehicles' arrival rate for $\mu = \lambda/2$. Curves for SCMS-PS and SCMS are overlapping. Points show the simulations results. Triangles refer to the SCMS case.



(a) $N = 5$ (b) $N = 10$

Fig. 5. Reject probability versus vehicles' arrival rate for $\mu = \lambda/2$. Curves for SCSM-PS and SCMS are overlapping. Points show the simulations results. Triangles refer to the SCMS case.

probability decreases as μ increases in the IP scenario because a new customers is always allowed to pick up an available car (even if not fully charged). Thus, the higher μ and the lower the probability that an incoming vehicle can not find an available parking space. On the other hand, in the FC scenario the blocking probability flattens out because, after a critical μ value, the probability to find a fully charged vehicle becomes low and an arriving customers are increasingly rejected (see also results in next section).

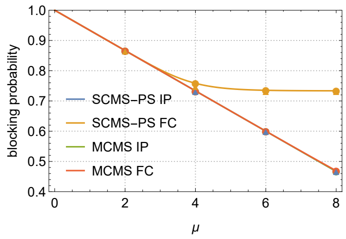
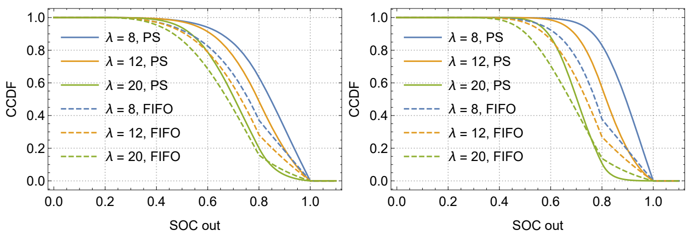


Fig. 4. Blocking probability versus customers' arrival rate for $\lambda = 15$ vehicles/hour and $N = 10$. Curves for SCMS-PS and SCMS are overlapping. Points show the simulations results. Triangles refer to the SCMS case.

2) Customer satisfaction: The second metric is the probability that a customer arriving at a station does not find an available vehicle, and thus it cannot access the mobility service. As explained in Section III-A, in case of IP scenario a customer is rejected only if the station is empty, while in the FC scenario also when there is not a fully-charge EV. Results in Figure 5 are obtained in the same configurations as for Figure 3. As expected, customer satisfaction would be low in the FC scenario, as the reject probability rapidly increases with λ. Again, SCMS-PS and SCMS models give the same performance as the power sharing features does not affect the average time that is needed to fully recharge vehicles.

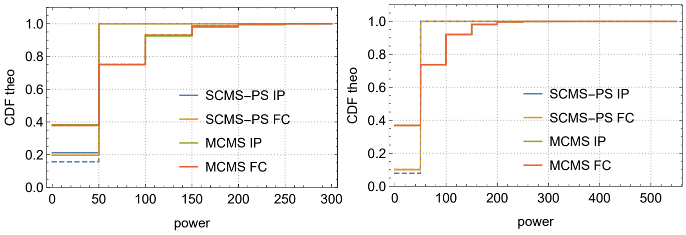
In an electric car sharing service customer satisfaction is also affected by the SoC of the rented vehicle. As a matter of fact, a low SoC may worse range anxiety, i.e., the driver's fear that the shared EV has insufficient battery energy to reach the intended destination. Clearly, this can happen only in the IP scenario, as in the FC scenario the customer is not allowed to rent non-fully charged vehicles. Figure 6 shows the Complementary Cumulative Distribution Function (CCDF) of the SoC of a vehicles at the time it leaves the charging station. We remind that the model thus not provide an analytical expression of this quantity, and the graphs report only simulation results. In all the considered scenarios we can observe that power sharing ensures a significant increase of the SoC of rented vehicles with respect to a conventional SCMS model. For example, for $N = 5$ and $\lambda = 8$ vehicles/hour, the probability that an EV leaves the station with a SOC at least equal to 80% is about 0.6 for SCMS-PS, while it is lower than 0.4 for simple SCMS. As discussed in Section III, queueing systems with processor sharing service discipline are more efficient than traditional FIFO discipline for jobs with small service requirements. Intuitively, while a server using FIFO discipline gets blocked for a large amount of time when a job with large service demand arrives, processor sharing discipline is able to serve all queued jobs simultaneously. This means that jobs with low service demands do not get blocked, if they arrive after a job with a large service demand. Note that the model in [11] assumes that each vehicle arrives at the charging station with the battery empty, which implies that all vehicles induce the same service demands. However, in real-world setting battery SoC of dropped off vehicles will be variable and this heterogeneity may have a notable impact on the performance of the charging station.



(a) $N = 5$ (b) $N = 10$

Fig. 6. CCDF of the SoC value for SAEVs leaving the charging station.

3) Power consumption: For the mobility operator's point of view the overall power consumption is a critical metric as it is related to the electricity costs. Furthermore, the peak charging load also has an impact on the cost because the higher the power peak and the larger the grid connectivity capacity that is required. Figure 7 shows the cumulative distribution function of the charging power drained from the power grid for the different charging models and scenarios when $\lambda = 8$ vehicles per hour. Once again, results confirm that accuracy of analytical predictions. Furthermore, we can observe that there is a clear advantage in using SCMS w/o power sharing since the charging peak load is bounded to 50kW, while in the MCMS model can be up to four times higher.



(a) $N = 5$ (b) $N = 10$

Fig. 7. CDF of charging power for $\lambda = 8$. The solid lines represent model predictions, the dashed lines simulation results.

To investigate more in depth the effect of power sharing on the energy performance of the charging station, Table II shows the power utilisation (see Equation 19), the peak power consumption and the average charging energy per day for a charging station using MCMS and SCMS-PS in the FC scenario. The results refer to a station with ten charging sockets, $\lambda = 15$ vehicles per hour and $\mu = \lambda/2$. The results clearly show that MCMS can put a strain on the power grid as it generates very high power peaks. Furthermore, the power utilisation (in brief, that ratio between average power consumption and power capacity) rapidly decreases as the power capacity increases. Interestingly, we can observe that if we consider a super-fast charging technology (i.e. $P = 150$ kW), SCMS-PS can provide the same energy per day of a MCMS station but with a much lower power peak. Furthermore, in this case the power utilisation is an order of magnitude higher with SMMS-PS than MCMS.

TABLE II
ENERGY PERFORMANCE FOR $\lambda = 15$ VEHICLES PER HOUR, $\mu = \lambda/2$, AND $N = 10$ IN THE FC SCENARIO.

P [kW]	Power Utilisation (%)		Max Power [kW]		Energy/Day [kWh]	
	MCMS	SCMS-PS	MCMS	SCMS-PS	MCMS	SCMS-PS
22	41.2%	99.8%	198	22	2184	527
50	18.6%	99.6%	450	50	2239	1196
150	6.2%	61.8%	900	150	2242	2226

V. Conclusion

In this work, we have investigated the performance of charging stations implementing a single charger but equipped with multiple charging sockets and a load management system to enable sharing the charging power with multiple EVs simultaneously. Differently from previous studies, we have focused on the emerging scenario of shared mobility system (e.g., an electrical car sharing service) in which customers randomly arrive at the charging station to pick up available shared EVs, thus leading to uncertain charging periods. With this goal in mind, we have developed a Markov-based model of the behaviour of the charging station, which provides accurate estimates on various metrics of interests, such as power utilisation the probability that there are not fully-recharged vehicles when a customer arrives. Such metrics are helpful to properly plan the size and energy capacity of a station. Furthermore, our results confirm the potential benefits of power sharing, in terms of higher power utilisation, reduced power peaks and increase of SoC of rented vehicles.

The model developed in this paper is intended as a first step towards a better understanding of the interplay between the efficiency of coordinated charging through power sharing, and shared mobility services. Future work consists in enhancing the proposed model to account for a generalised pattern of customer's arrival . Furthermore, we plan to extend the model formulation to describe the performance of a network of charging stations, in order to determine the optimal locations and sizes of the charging stations needed to serve the charging demands of a shared fleet in a real case study.

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